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R. Gatto: EFFECTS OF VIRTUAL VECTOR MESONS IN  $e^+e^-$   
COLLISIONS.

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## Effects of Virtual Vector Mesons in $e^+e^-$ Collisions.

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The main object of this note is an estimate of the vacuum-polarization effects due to virtual vector mesons in high-energy electron-positron reactions. We also comment shortly on the resonance contributions arising from production and decay of vector mesons in such reactions.

We estimate that at energies near the meson mass the vacuum-polarization effects modify the cross-section formula, for reactions going through the one-photon channel, by a factor  $\mathcal{F}(E)$  given in eq. (14), where  $\Gamma$  is the meson decay width,  $2E$  is the total incident energy,  $M$  is the meson mass,  $\alpha=1/137$  and  $b$  is the branching ratio, divided by  $\alpha^2$ , for decay of the meson into  $e^+ + e^-$ . Unfortunately in the most important case of effects due to the  $\omega^0$  mesons the parameters  $\Gamma$  and  $b$  are very little known at present. Our numerical estimates are based on calculations due to GELL-MANN, SHARP and WAGNER, which make predictions for such parameters <sup>(1)</sup>. If such predictions will be modified our numerical estimates will consequently be changed.

We write the modified Feynman propagator for the photon in the form

$$(1) \quad D'_{\mu\nu}(k) = \delta_{\mu\nu} \frac{1}{k^2} + \left( \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \int_0^\infty \frac{da \pi(-a)}{a(k^2 + a - i\varepsilon)},$$

where the quantity  $\pi$  is defined by

$$(2) \quad \pi(k^2) = -\frac{(2\pi)^3}{3k^2} \sum_{p(z)=k} \langle 0 | j_\nu(0) | z \rangle \langle z | j_\nu(0) | 0 \rangle.$$

In (2)  $j_\nu$  is the operator for the electromagnetic current and the summation is extended over all physical states  $z$  with total four-momentum  $k$ . Equations (1) and (2) can be used to calculate the modifications of the photon propagator due to

<sup>(1)</sup> M. GELL-MANN, D. SHARP and W. G. WAGNER: *Phys. Rev. Lett.*, **8**, 261 (1962).

virtual strong interacting particles. We consider the annihilation process into a set of states  $z$

$$(3) \quad e^+ + e^- \rightarrow z,$$

(for instance,  $z$  is the set of all  $2\pi$  states, or the set of all  $2\pi$  and  $3\pi$  states, or the set of all nucleon-antinucleon states, etc.) and call  $\sigma_z(E)$  the cross-section for (3) when the incident electron has energy  $E$  in the center-of-mass system (the total four-momentum squared of  $e^+ + e^-$  is then  $(p^{(+)} + p^{(-)})^2 = -4E^2$ ). Similarly we call  $\pi_z(-4E^2)$  the contribution to (2) from the set of states  $z$ , taken at  $K^2 = -4E^2$ . The two quantities  $\sigma_z$  and  $\pi_z$  are related by

$$(4) \quad \pi_z(-4E^2) = \frac{E^2}{\pi^2 \alpha} \sigma_z(E),$$

where  $\alpha = 1/137$ .

Equation (4) follows formally by comparing (2) with the expression for  $\sigma_z$  as derived from the amplitude

$$(5) \quad \langle z | S | e^+ e^- \rangle = \frac{2\pi e}{k^2} (\bar{v} \gamma_\nu u) \langle z | j_\nu(0) | 0 \rangle \delta(p^{(+)} + p^{(-)} - p^{(z)}),$$

for  $e^+ + e^- \rightarrow z$  at lowest electromagnetic order. In eq. (5),  $k = p^{(+)} + p^{(-)}$  and  $u$  and  $v$  are spinors for  $e^-$  and  $e^+$  respectively. The cross-section  $\sigma_z$  can then be written as

$$(6) \quad \sigma_z(E) = -\frac{(2\pi)^5 \alpha}{16E^4} \frac{1}{2} \left( \frac{p_m^{(+)} p_n^{(+)}}{|\mathbf{p}|^2} - \delta_{mn} \right) \sum_{p^{(z)}=k} \langle 0 | j_n(0) | z \rangle \langle z | j_n(0) | 0 \rangle,$$

where  $m, n = 1, 2, 3$  and the condition

$$(7) \quad K_\mu \langle z | j_\mu(0) | 0 \rangle = 0,$$

allows one to express  $\sigma_z$  in terms of  $\pi_z(k^2)$  as given by (2).

The amplitude for  $e^+e^-$  annihilation into a final state  $f$ , including lowest e.m. order vacuum polarization effects due to strong interacting particles, can be written as

$$(8) \quad \langle f | S | e^+ e^- \rangle = \left[ 1 + k^2 \int_0^\infty \frac{da \pi(-a)}{a(k^2 + a - i\varepsilon)} \right] \frac{2\pi e}{k^2} (\bar{v} \gamma_\nu u) \langle f | j_\nu(0) | 0 \rangle \delta(p^{(+)} + p^{(-)} - p^{(f)}),$$

where  $\pi(-a)$  is calculated using (2) and summing over the intermediate states  $z$  of strong interacting particles. In deriving eq. (8) one has to take into account the transversality condition (7). From eq. (8) one sees that the cross-section  $d\sigma$  is modified, by the inclusion of vacuum polarization effects due to strong interacting particles at lowest electromagnetic order, by a multiplicative factor

$$(9) \quad \mathcal{F}(E) = \left| 1 - 4E^2 \int_0^\infty \frac{da \pi(-a)}{a(-4E^2 + a - i\varepsilon)} \right|^2.$$

From the relation (4) we see that the vacuum-polarization factor  $\mathcal{F}(E)$  can be evaluated from the experimental values  $\sigma$  of the annihilation cross-section into strong interacting particles. The cross-section  $\sigma$  is needed however for a presumably large range of the incident energy, as the calculation of  $\mathcal{F}(E)$  implies, according to (9), an integration over the variable  $a$ , which is the square of the total incident energy. Unfortunately no experimental data are available at present on the annihilation cross-section of  $e^+e^-$  into strong interacting particles. We can however easily estimate the contribution to  $\mathcal{F}(E)$  from a strong resonant state having a width  $\Gamma$  and a branching ratio  $B$  for decay into  $e^+e^-$ . Such a resonant state must, of course, have the appropriate quantum numbers to couple to the  $e^+e^-$  system at lowest electromagnetic order, namely (total angular momentum)=1, (parity)=-1, (charge conjugation number)=-1, (isotopic spin)=0 or 1, and zero strangeness. Of the known meson resonances,  $\rho^0$  and  $\omega^0$  have these quantum numbers. The set of states  $z$  is the set of decay states of the resonance and we shall describe the amplitudes for  $e^+e^- \rightarrow z$  by expressions having the right analytical behaviour and correctly normalized at the resonance. In the neighbourhood of the resonance the resonant cross-section for annihilation into a final channel  $f$  will be approximated by a Breit-Wigner formula

$$(10) \quad \sigma_f(E) = \pi\lambda^2 \frac{3}{4} BB_f \frac{\Gamma^2}{(2E - M)^2 + \frac{1}{4}\Gamma^2},$$

where  $B_f$  is the branching ratio for decay of the resonance into  $f$  and  $M$  is the mass of the resonance. We then introduce an amplitude  $T(a)$

$$(11) \quad T(a) = \frac{12\pi}{M} B \frac{\Gamma(a)}{M^2 - a - iM\Gamma(a)},$$

where  $\Gamma(a)$  is such that  $T(a)$  has the correct analytical properties to describe the annihilation process  $e^+e^- \rightarrow f$ , it is slowly varying near  $M^2$  and  $\Gamma(M^2) = \Gamma$ .

We assume that

$$(12) \quad T(-k^2) = \frac{1}{\pi} \int \frac{\text{Im } T(a) da}{a + k^2 - i\epsilon}.$$

The imaginary part of  $T(a)$  near the resonance can be approximated as

$$(13) \quad \text{Im } T(a) \simeq \frac{3\pi}{a} B \frac{\Gamma^2}{(M - \sqrt{a})^2 + \Gamma^2/4} = \sum_f \sigma_f(E),$$

with  $a = 4E^2$ . Making use of (4) and (9) we thus find, near the resonance,

$$(14) \quad \mathcal{F}(E) \simeq \left| 1 - \frac{3}{2} \alpha b \frac{\Gamma}{M - 2E - i(\Gamma/2)} \right|^2,$$

where we have written

$$(15) \quad B = \alpha^2 b,$$

putting into evidence the factor  $\alpha^2$  in the decay rate into  $e^+e^-$ .

When comparing with experiments one has to take into account explicitly the experimental energy resolution  $\Delta E$ . Let us first consider the case  $\Delta E \gg \Gamma$ , such that an average over the resonance region is really observed. The measured average is

$$(16) \quad \frac{1}{\Delta E} \int_{\frac{1}{2}(M-\Delta E)}^{\frac{1}{2}(M+\Delta E)} d\sigma dE.$$

We write

$$(17) \quad d\sigma = \mathcal{F}(E) d\sigma_0,$$

where  $d\sigma_0$  is the cross-section without the inclusion of the resonance vacuum polarization. We find, inserting the expression (14) for  $\mathcal{F}(E)$

$$(18) \quad \overline{d\sigma} = \left[ 1 + \frac{\Gamma}{\Delta E} (1 + 9\alpha^2 b^2) \frac{1}{4} \int_{-2\Delta E/\Gamma}^{2\Delta E/\Gamma} dy \frac{1}{y^2 + 1} \right] d\sigma_0 \simeq \left[ 1 + \frac{\Gamma}{\Delta E} (1 + 9\alpha^2 b^2) \frac{\pi}{4} \right] d\sigma_0.$$

The term proportional to  $\alpha$ , odd about the resonance, has disappeared, and only an effect proportional to  $\alpha^2$  in the correction factor remains. Such a residual term is of order  $e^8$  in the cross-section and at that order many other radiative corrections are important. We apply eq. (18) to the case of  $e^+e^-$  annihilation at an energy around the  $\omega$  mass, using the values proposed by GELL-MANN, SHARP and WAGNER for the  $\omega$  width  $\Gamma$  and for the branching ratio of  $\omega$  into  $e^+e^-$ . The width  $\Gamma$  estimated by these authors is  $\Gamma \simeq 0.5$  MeV and the branching ratio  $B \simeq 4.6 \cdot 10^{-3}$ . We obtain

$$(19) \quad \overline{d\sigma} \simeq \left[ 1 + \frac{1.8}{(\Delta E \text{ in MeV})} \right] d\sigma_0,$$

which is a very big correction for  $\Delta E$  of the order of some MeV. Other radiative corrections are presumably much smaller.

The attainable energy resolution  $\Delta E$  in experiments with electron-positron storage rings will presumably be very small. In the project Adone, under development in Frascati, the energy resolution  $\Delta(2E)$  expected around 800 MeV, with a storage ring having maximum energy  $2E_{\max} = 3$  GeV, is  $\Delta(2E) = 0.15$  MeV <sup>(2)</sup>.

With such a small energy resolution one can directly compare with  $\mathcal{F}(E)$  as given by (13). In terms of  $y = 2(2E - M)/\Gamma$  and  $\varrho = 3\alpha b$  one can write

$$(20) \quad \mathcal{F}(E) = \frac{(y + \varrho)^2 + 1}{y^2 + 1}.$$

The maximum of  $\mathcal{F}$  occurs at  $y_{\max} = -(\varrho/2) + [(\varrho/2)^2 + 1]^{1/2}$ , corresponding to an energy

$$(21) \quad 2E_{\max} = M + \frac{\Gamma}{2} \left\{ \left[ \left( \frac{3}{2} \alpha b^2 \right) + 1 \right]^{1/2} - \frac{3}{2} \alpha b \right\},$$

<sup>(2)</sup> Private communication from C. BERNARDINI.

and it is given by

$$(22) \quad \mathcal{F}(E_{\max}) = 1 + 3\alpha b \left\{ \left[ \left( \frac{3}{2} \alpha b \right)^2 + 1 \right]^{\frac{1}{2}} + \frac{3}{2} \alpha b \right\}.$$

With the values of  $\Gamma$  and  $b = \alpha^{-2} B$  used before,  $\Gamma = 0.5$  MeV and  $B = 4.6 \cdot 10^{-3}$  we find

$$(23) \quad 2E_{\max} = M + 0.11 \text{ MeV},$$

$$(24) \quad \mathcal{F}(E_{\max}) = 5.5,$$

The last result shows the possibility of a very big increases of the cross-sections, at an energy close to  $M$ , from the vacuum polarization effects due to the  $\omega$ -meson. Although the formulae we have used are general and essentially rigorous, the numerical estimates (19), (23) and (24) have been obtained by using uncritically the values  $\Gamma$ , for the  $\omega$  decay width, and  $B$ , for the  $\omega$  branching ratio into  $e^+e^-$ , as given by GELL-MANN, SHARP and WAGNER. If such values turn out to be wrong our predictions must correspondingly be modified.

The basis for the estimate given by Gell-Mann, Sharp and Wagner is a calculation of the  $\pi^0$  decay rate with a model in which the  $\pi^0$  disintegrates virtually into a  $\rho$  and an  $\omega$ . Such a model seems however to be unable to reproduce the measured negative value of the coefficient  $a$  in the expansion.

$$(25) \quad G(k^2) = G(0) \left( (1 + ak^2/m_\pi^2 + \dots) \right),$$

of the  $\pi^0$  form-factor  $G$  in powers of the momentum transfer  $k^2$  (<sup>3,4</sup>). A model consistent with the measured values of  $a$  requires, in addition to the contribution from  $\rho + \omega$  states, a large contribution from higher mass states, that largely cancels the  $\rho + \omega$  contribution in the  $\pi^0$  decay rate (<sup>5</sup>). The coupling of  $\omega$  to  $\rho + \pi$ , as estimated from the experimental  $\pi^0$  lifetime, is then much larger than with the model assuming only intermediate  $\rho + \omega$  states. If such a coupling is responsible for the dominant  $\omega$ -decay into  $3\pi$ , the width for such a decay mode is correspondingly increased. The branching ratio  $B$  on the other hand becomes smaller. If this is indeed the situation the big effect we found, eq. (24), may almost entirely get washed out. In any case electron-positron colliding beams will be able to answer very directly such a question and similar ones that may occur. In the meantime more accurate measurements of the parameter  $a$  in (25) would be very useful.

Vacuum polarization effects from the  $\rho^0$  will be much less important because of the small branching ratio for  $\rho^0$  decay into two electrons. A detailed calculation was carried out by BROWN and CALOGERO, who show that the effect is small (<sup>6</sup>).

Of course, the most apparent effect of vector mesons in  $e^+e^-$  collisions will be the occurrence of resonance peaks in annihilation processes into final channels that are more strongly coupled to the vector mesons. A general discussion of such resonant contributions has already been given (<sup>7</sup>). A big resonant contribution is

(<sup>3</sup>) N. P. SAMIOS: *Phys. Rev.*, **121**, 275 (1961).

(<sup>4</sup>) H. KOBRAK: *Nuovo Cimento*, **20**, 1115 (1961).

(<sup>5</sup>) D. GEFFEN: *Phys. Rev.*, **123**, 374 (1962).

(<sup>6</sup>) L. M. BROWN and F. CALOGERO: *Phys. Rev.*, **120**, 653 (1960).

(<sup>7</sup>) R. GATTO: *Proc. of Intern. Conf. on Theoretical Aspects of Very High Energy Phenomena*, CERN, 61-22; August; N. CABIBBO and R. GATTO: *Phys. Rev.*, **124**, 1577 (1961).

expected, for instance, in  $e^+ + e^- \rightarrow \pi^+ + \pi^-$ , going through intermediate  $\rho^0$ . Similarly, at an energy  $2E$  close to the  $\omega$ -mass, resonant peaks will occur in the processes.

$$(26) \quad e^+ + e^- \rightarrow \omega^0 \rightarrow \pi^+ + \pi^- + \pi^0,$$

$$(27) \quad e^+ + e^- \rightarrow \omega^0 \rightarrow \pi^0 + \gamma,$$

$$(28) \quad e^+ + e^- \rightarrow \omega^0 \rightarrow \pi^+ + \pi^-,$$

all proceeding through intermediate  $\omega^0$ . The cross-sections for (26), (27) and (28) can be predicted from the values of the decay width calculated by GELL-MANN, SHARP and WAGNER. At an energy corresponding to the  $\omega$  mass we find for the resonant cross-section the peak values

$$(29) \quad \sigma_{\text{res}}(e^+e^- \rightarrow 3\pi) = 0.85 \cdot 10^{-28} \text{ cm}^2,$$

$$(30) \quad \sigma_{\text{res}}(e^+e^- \rightarrow \pi^0\gamma) = 1.5 \cdot 10^{-29} \text{ cm}^2,$$

$$(31) \quad \sigma_{\text{res}}(e^+e^- \rightarrow 2\pi) = 3.6 \cdot 10^{-30} \text{ cm}^2.$$

The resonance width is essentially the  $\omega^0$  width, supposed to be very narrow. A measurement of the cross-sections (29), (30) and (31) will then require a very accurate energy resolution for the colliding beams, which seems to be possible with storage rings. The values (29), (30) and (31) for the resonant cross-sections are unbelievably large. We can compare these cross-sections with the cross-section for  $e^+ + e^- \rightarrow \mu^+ + \mu^-$ , approximately given by  $\sigma(e^+e^- \rightarrow \mu^+\mu^-) \simeq \frac{1}{3}\pi\alpha^2\lambda^2$ . [One should note that at an energy close to the  $\omega^0$  mass this formula does not strictly apply if the vacuum polarization effects that we discussed before, are relevant.] The cross-section (29) is about 600 times larger than  $\frac{1}{3}\pi\alpha^2\lambda^2$ ; the cross-section (30) about 100 times larger; and the cross-section (31) about 25 times larger. Note also that, for instance,  $e^+ + e^- \rightarrow \pi^0 + \gamma$  is of higher electromagnetic order than  $e^+ + e^- \rightarrow \mu^+ + \mu^-$ . Precisely (26) is of the order  $\alpha^2$  (the same as  $e^+e^- \rightarrow \mu^+\mu^-$ ), whereas (27) is of order  $\alpha^3$ , and (28) of the order  $\alpha^4$ . In fact the  $\omega^0$  decay into  $2\pi$  violates isotopic spin selection rule. One would then be faced by a very peculiar situation. Again we stress that the above figures are entirely based on the estimates by GELL-MANN, SHARP and WAGNER of the  $\omega^0$  widths, and must be modified if such estimates are changed. For instance, if Geffen's analysis is right the resonant cross-sections (29), (30) and (31) become much smaller. In such a case the cross-section (29) becomes  $\sigma_{\text{res}}(e^+e^- \rightarrow 3\pi) = 0.24 \cdot 10^{-29} \text{ cm}^2$ , which is still however very large.